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Research note

Optimal trajectory planning of wheeled mobile manipulators in cluttered environments using potential functions

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Abstract This paper presents an advanced methodology for collision-free trajectory planning of wheeled mobile manipulators in obstructed environments by means of potential functions. In the presented method, all mobile manipulator parts and environmental obstacles are modeled as ellipsoids. Due to collision avoidance, the ellipsoid equations are expressed in a reference coordinate system and the corresponding dimensionless potential functions are defined. Then, the trajectory planning of a spatial mobile robot in cluttered environment is performed, employing optimal control theory. Beyond simplicity and novelty of the proposed method, depletion of prior methods is rectified, which lead to excessive computation and singularity during process. Also, a number of simulations and experiments for Scout mobile manipulator are carried out, which illustrate the power and efficiency of the proposed method.

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1. Introduction

Wheeled mobile manipulators operate in a wide range of environments like factories and houses including variant types of obstacles. So, mobile robots must be planned to maneuver their trajectory in cluttered environments without colliding to the obstacles. An extensive review of some conventional path planning methods in the presence of the obstacles is studied in [1]. Here in, analytical methods [2], as well as graphical algorithms [3], are discussed thoroughly. The trajectory planning of a point mobile robot in the presence of a point obstacle is studied in [4] based on fuzzy behavior method. Papadopoulos et al. [5] proposed a strategy for trajectory planning of mobile manipulators in the presence of multiple obstacles. They used a polynomial function to describe the mobile robot trajectory. Although obstacle avoidance has been allowed by increasing the order of the polynomials, a weak point of this procedure

is limiting the solution to a fixed-order polynomial. Gonzalez et al. [6] used potential field method for motion planning of mobile robots in the presence of circular obstacles. They only considered the kinematic model of mobile robot, which is not a proper modeling for path planning of manipulators, and torque capacity of robot's actuators must be taken into account. Dierks and Jagannathan [7] presented the collision-free motion planning of a planar mobile robot by means of neural network control strategy in a two-dimensional obstructed space. Toronto et al. [8] used spheres for collision detection of spatial objects. Modeling of obstacles, as the spheres, reduces the complexity of geometrical representations, but it does not have enough precision for modeling of mobile manipulator arms. Furthermore, a set of spheres with variant radii is used in [9] to model environmental objects such as mobile robot parts and obstacles. In the proposed procedure, determining the intersection of all colliding spheres is a challenging issue, especially when mobile manipulator and obstacle are close to one another. Lei et al. [10] employed the vertices of convex objects to avoid the collision between two polygons. The computation cost of the algorithm is nearly a linear function of total numbers of the object edges. In addition, Ju et al. [11] proposed a numerical method for collision detection of spatial objects according to the ellipsoid models of objects. However, their algorithm did not depend on the numbers of objects vertices, but computing the free margin between two ellipsoids leads to a complicated iterative algorithm with expense calculation.

Most of the mentioned papers have only dealt with collision-free trajectory planning of mobile robots, and also considered simple models of objects in the cluttered environments. Indeed,

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Nomenclature

O	Center point of the global coordinate system
G	Center point of the local coordinate system
\bar{X}_0	Distance from O to G
A	Transformation matrix
p_i	Position of the i th obstacle in local coordinate system
P_i	Position of the i th obstacle in global coordinate system
a, b, c	Equatorial radii and polar radius of the ellipsoid, respectively
$2l, 2k, 2h$	Length, width and height of the rectangular cuboid, respectively
r_{ob_i}	Radius of the i th sphere obstacle
$a_{ob_i}, b_{ob_i}, c_{ob_i}$	Equatorial radii and polar radius of the ellipsoidal obstacle, respectively
$C(X, Y, Z)$	Position of mass center of platform in global coordinates
Q	Intersection of the symmetry axis of base with the driving wheel axis
r	Radius of each wheel
d	Distance from Q to C
e	Distance from Q to center point of each wheel
ϕ	Heading angle of the platform
l_1, l_2	Length of the first and second links of mobile manipulator, respectively
θ_r, θ_l	Angular displacement of the right and left wheels, respectively
θ_1, θ_2	Angular displacement of the first and second links, respectively
J_w	Moment of inertia of each wheel and actuator about the wheel axis
J_l	Moment of inertia of each wheel and actuator about the wheel diameter
J_0	Moment of inertia of the mobile platform about a vertical axis
J_1, J_2	Moment of inertia of the first and second links about a vertical axis
m_w, m_p	Mass of each wheel and payload
m_1, m_2	Mass of the first and second links, respectively
W, R	Weighting matrices of the state and control vectors, respectively
$w_{ob_{ib}}$	Weighting coefficient related to the potential function between the i th obstacle and mobile platform
$w_{ob_{il_1}}$	Weighting coefficient related to the potential function between the i th obstacle and first link of the mobile manipulator
$w_{ob_{il_2}}$	Weighting coefficient related to the potential function between the i th obstacle and second link of the mobile manipulator
t_f	Final time of the mobile robot motion.

optimal motion planning is one of the most appealing tasks in robotics manipulation, and has been studied by some authors. Korayem et al. [12] presented an analytical method for optimal path planning of a flexible manipulator based on iterative linear programming. Using the iterative linear programming cannot result in proper convergence to the answer, especially for nonlinear systems having high-speed motion. Also, the path optimization of mobile manipulators is studied in [13],

but no obstacles are assumed in robot's workspace. Chettibi et al. [14] considered the optimal path planning of planar manipulators, and modeled robot links and obstacles as a set of balls. Their method exceeds numerical computation and makes it impractical, especially for systems with a large degree of freedom, such as the wheeled mobile manipulator. Korayem et al. [15] proposed an algorithm to determine the maximum load carrying capacity of mobile manipulators, considering tip over stability in the presence of obstacles. They only modeled the environmental obstacles as circles, and the robot links as lines. Beside the triviality of obstacles representation, modeling of robot arms, as the infinite lines, results in singularity during the process of the path generation.

Therefore, the literature shows that there is a need to reinvestigate a comprehensive method for the trajectory planning of mobile manipulators in obstructed environments, which comprises a simple and enough accurate procedure for collision avoidance among spatial objects. In this paper, a novel method for trajectory planning of wheeled mobile is presented using potential functions terms. Due to the collision detection, environmental obstacles and mobile manipulator parts (the mobile platform and the arms) are generally modeled as ellipsoids. Regarding the transformation between the global and local coordinate systems, the ellipsoids equations are expressed in the reference coordinate system, and the corresponding potential functions are defined for the obstacle avoidance of mobile manipulator. The nonlinear dynamics equations of mobile robot are derived, and the optimal control theory is employed for trajectory planning of mobile robot in cluttered environment. The simplicity and precedence of the formulation of algorithm over the prior methods are discussed, and simulations and experimental results for Scout mobile manipulator are performed, which demonstrate the capability of the proposed method. The paper is organized as follows. In Section 2, a general scheme of the method for optimal trajectory planning is presented. Also, the modeling of spatial objects with enclosed ellipsoids is considered and some new potential functions for collision avoidance are defined. In Section 3, the dynamics equations of Scout mobile robot are derived. To verify the proposed method, optimal trajectory planning of Scout mobile manipulator is simulated and compared to the experimental results in Section 4. Finally, the paper is concluded by a brief summary in Section 5.

2. General scheme of the optimal trajectory planning in the cluttered environments

Optimal trajectory planning of wheeled mobile robots in cluttered environment is a complex and important task. In this paper, the general scheme of the method can be arranged in the following order. At first, the dimensions of mobile manipulator parts and obstacles are obtained. The recognition of obstacle shapes as cloud points can be performed via some conventional methods such as image processing (Environment Recognition). Then, environmental obstacles and mobile manipulators parts are enclosed by ellipsoids using optimization methods (Environment Modeling). In the next stage, the appropriate potential functions are formulated according to ellipsoid models of spatial objects (Obstacle Avoidance). Then, the initial and final states of robot motion are determined, and trajectory planning and path optimization of non-holonomic mobile manipulator are done (Optimal Trajectory Planning). Figure 1 represents the general scheme of the method.

2.1. Trajectory planning and obstacle avoidance formulation

The problem of trajectory optimization of wheeled mobile manipulators in the obstructed environments is dealt with

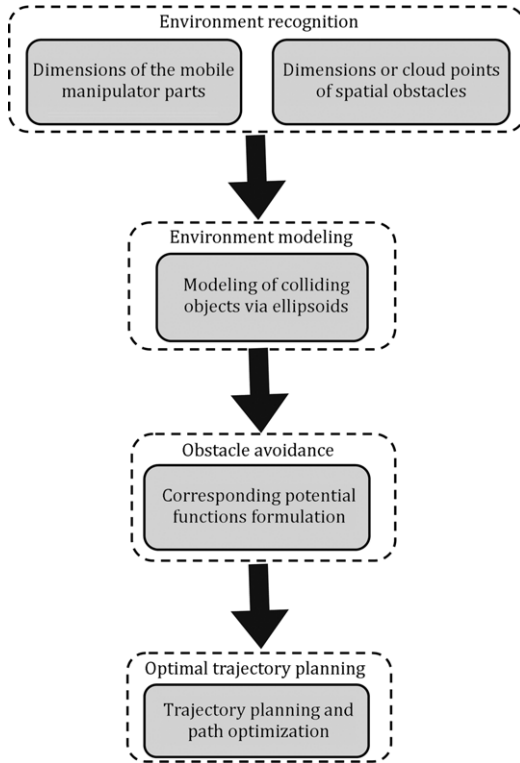


Figure 1: General scheme of the trajectory planning in cluttered environment.

finding the optimum values of their generalized coordinates and control inputs. The optimal trajectory planning can be suitably formulated as an optimal control problem. The dynamics equations of mobile robot in the state space form, $\dot{X} = F(X(t), U(t))$, are presumed as the constraints of optimal control problem, and is aimed to determine the optimal state vector, X^* , and the optimal control vector, U^* , such that the following objective function can be minimized [16]:

$$J(X, U) = \int_{t_0}^{t_f} L(X(t), U(t), t) dt. \quad (1)$$

The objective function is presumed a function of the actuator velocities and torques in addition to the potential function terms:

$$L(X(t), U(t)) = \frac{1}{2} \|X\|_W^2 + \frac{1}{2} \|U\|_R^2 + \frac{1}{2} \|L_i\|_{w_{objj}}^2, \quad (2)$$

where $\|X\|_W^2$ is the generalized squared norm of the state vector X with respect to a state weighting matrix W , and $\|U\|_R^2$ is the generalized squared norm of the control vector U with respect to a control weighting matrix R . The parameter $\|L_i\|_{w_{objj}}^2$ refers to the potential function between the i th obstacle and the j th part of mobile manipulator.

By implementing the indirect solution of the optimal control problem, necessary conditions for optimal trajectory planning of mobile manipulator can be achieved. It must be mentioned that the indirect method of optimal control problem does not require linearizing dynamics equations, and it is known as an appropriate method for path planning of system with a large degree of freedom. To get the necessary conditions of optimality of the problem, the Hamiltonian function is defined as:

$$H(X, U, \psi, t) = L(X, U) + \psi^T(t)F(X, U).$$

The conditions of optimal trajectory planning lead to a two-point boundary value problem with the following equations:

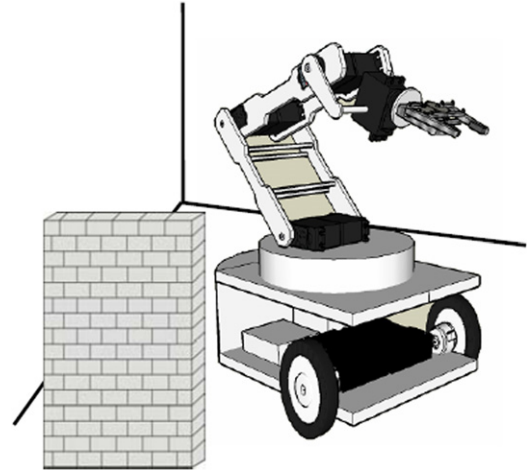


Figure 2: Spatial mobile robot in a real environment.

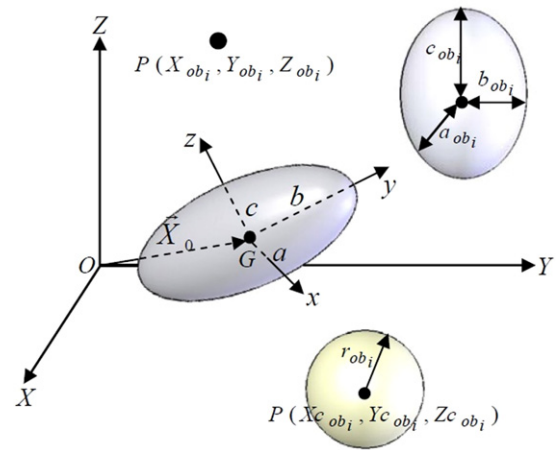


Figure 3: Enclosed ellipsoid in the presence of the general obstacles.

$$\dot{X}^*(t) = \frac{\partial H}{\partial \psi}(X^*, U^*, \psi^*, t), \quad (3)$$

$$\dot{\psi}^*(t) = -\frac{\partial H}{\partial X}(X^*, U^*, \psi^*, t), \quad (4)$$

$$H(X^*, U^*, \psi^*, t) \leq H(X^*, \psi^*, \bar{U}, t), \quad (5)$$

where the symbol $(*)$ refers to the extremals of $X(t)$, $U(t)$ and $\psi(t)$. The established two-point boundary value problem is solved in MATLAB with `bvp4c` function.

But the most important issue is obstacle avoidance of mobile robot, which is implied by the potential function terms. In this paper, the environmental objects are generally modeled as ellipsoids, and due to obstacle avoidance of mobile manipulator in cluttered environment, the new dimensionless potential functions are defined.

Furthermore, it should be mentioned that a lot of ellipsoids can be enclosed to the convex object, but fitting the best ellipsoid to the object can be treated as an optimization problem. Moreover, mobile robot parts (the platform and the arms), walls, etc. are usually rectangular objects, and it is obvious that the rectangular cuboid must be enclosed with the optimal ellipsoid. Figure 2 shows a non-holonomic spatial mobile manipulator in a real environment, and Figure 3 depicts the enclosed ellipsoid to mobile manipulator parts in the presence of some common obstacles.

In addition, for obstacle avoidance of colliding objects, potential function is applied to performance index, where it is a function of the dimensionless parameter, d_{ij} , and can be written as:

$$\|L_i\|_{w_{obij}}^2 = w_{obij} \frac{1}{d_{ij}^2}. \quad (6)$$

The value of the potential function is increased when the mobile robots move closer to the environmental obstacles.

For determining the parameter, d_{ij} , suppose that each robot's part is modeled by ellipsoid, and a point obstacle is in the robot's workspace (Figure 3). In fact, the distance between point obstacle and enclosed ellipsoid can be computed via optimization method, which leads to a complex equation. But on the other point of view, the relative position of the point obstacle with respect to the ellipsoid can be easily described according to its describing equation. If the equation of the ellipsoid is considered in the local coordinate system, xyz , attached to its center, for the collision avoidance between the point obstacle with the local coordinates $p_i(x_{obi}, y_{obi}, z_{obi})$, and the enclosed ellipsoid, the parameter d_{ij} is defined as follows:

$$d_{ij} = \left(\frac{x_{obi}^2}{a^2} + \frac{y_{obi}^2}{b^2} + \frac{z_{obi}^2}{c^2} - 1 \right)^{\frac{1}{2}}, \quad (7)$$

where d_{ij} is a dimensionless parameter and implies the obstacle avoidance of ellipsoid, if it has the real positive value.

Since the orientation of ellipsoid is variant in three-dimensional space, and the position of point obstacle is expressed in the reference coordinate system, the simple Eq. (7) is not appropriate for collision avoidance of ellipsoid in the presence of point obstacle in global coordinate system. So, the transformation between local and global coordinate must be performed, and a new formulation of d_{ij} should be stated. Suppose that vector $\vec{x} = [x \ y \ z]^T$ represents the equation of ellipsoid in local coordinate system and vector $\vec{X} = [X \ Y \ Z]^T$ shows the global representation of ellipsoid. The vectors are related to each other via the transformation matrix A :

$$\vec{x} = A^{-1}\vec{X} = \begin{bmatrix} f(X, Y, Z) \\ g(X, Y, Z) \\ h(X, Y, Z) \end{bmatrix}. \quad (8)$$

Therefore, according to Eqs. (7) and (8), for collision avoidance between point obstacle and enclosed ellipsoid, the parameter d_{ij} in global coordinate system is given by:

$$d_{ij} = \left(\frac{f^2(P_i)}{a^2} + \frac{g^2(P_i)}{b^2} + \frac{h^2(P_i)}{c^2} - 1 \right)^{\frac{1}{2}}, \quad (9)$$

where functions f , g and h must be evaluated in the obstacle coordinate $P(X_{obi}, Y_{obi}, Z_{obi})$.

Eq. (9) implies the collision avoidance between point obstacle and enclosed ellipsoid in global coordinate system and can be suitably substituted in potential function formulation.

Also, by continuing a similar procedure for collision avoidance between fitting ellipsoids and spherical or ellipsoidal obstacles, the parameter d_{ij} is derived as Eq. (9). But, the parametric equations of spherical or ellipsoidal obstacles must be substituted in potential function formulation.

3. Dynamic model of Scout mobile manipulator

In the previous section, the general scheme of trajectory planning of mobile manipulator in the presence of obstacles is



Figure 4a: Picture of a Scout mobile robot.

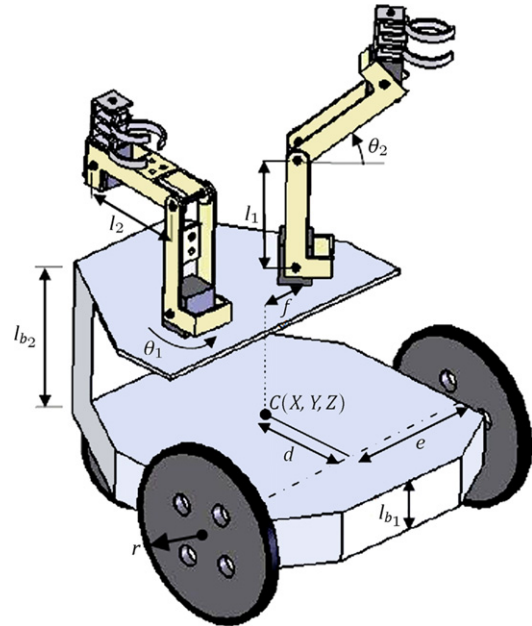


Figure 4b: Schematic of a Scout mobile robot.

discussed based on the new definition of potential functions. But in order to achieve obstacles avoidance of wheeled mobile manipulators, the full dynamic model of mobile robot must be taken into account, by considering all interactions of mobile platform and mounted manipulator. A Scout mobile robot has two spatial arms and a mobile platform with two driving wheels, which are independently driven by two actuators. The picture and the schematic of the Scout robot are shown in Figures 4a and 4b, respectively.

The generalized coordinates are defined as vector $q = [X \ Y \ \phi \ \theta_r \ \theta_l \ \theta_1 \ \theta_2]^T$, and according to the Lagrange

equations of the non-holonomic mobile manipulator, the dynamics equations of the Scout robot can be generally presented as follows:

$$M(q)\ddot{q} + C(q, \dot{q}) + G(q) = BU + D^T \Omega, \quad (10)$$

where $M_{7 \times 7}$ is the inertia matrix, and $C_{7 \times 1}$ is the vector of Coriolis and centrifugal forces. The vector $G_{7 \times 1}$ describes the gravity effects and $U_{4 \times 1}$ is the generalized force inserted into the actuator. The vector $\Omega_{3 \times 1}$ is referred to the Lagrange multipliers term. In addition, the matrix $D_{3 \times 7}$ represents the non-holonomic constraints of mobile platform and is equal to:

$$D = \begin{bmatrix} -\sin \phi & \cos \phi & -d & 0 & 0 & 0 & 0 \\ -\cos \phi & -\sin \phi & -b & r & 0 & 0 & 0 \\ -\cos \phi & -\sin \phi & b & 0 & r & 0 & 0 \end{bmatrix}. \quad (11)$$

By representing the velocity vector as:

$$v = [\dot{\theta}_r \quad \dot{\theta}_l \quad \dot{\theta}_1 \quad \dot{\theta}_2]^T,$$

and considering matrix S whose columns are in the null space of D matrix, the state-space form of the dynamic model of a Scout mobile manipulator can be summarized as follows:

$$\dot{X} = \begin{bmatrix} S v \\ (S^T M S)^{-1} (-S^T M \dot{S} v - S^T (C + G)) \end{bmatrix} + \begin{bmatrix} 0 \\ B(S^T M S)^{-1} \end{bmatrix} U. \quad (12)$$

3.1. Modeling of Scout robot parts

A Scout is a wheeled mobile manipulator with a non-holonomic platform and two spatial arms. To model mobile base and links with the enclosed ellipsoids, the transformation between reference coordinate system and local coordinate system attached to the robot parts should be done. For the collision avoidance between Scout mobile base and the obstacles, the vector (x_b, y_b, z_b) is supposed as the local equation of fitted ellipsoid to the robot's platform and the vector (X_b, Y_b, Z_b) is the global equation of ellipsoid, which are related via the transformation matrix A_b :

$$A_b^{-1} = \begin{bmatrix} -R_b^T & -R_b^T \vec{X}_{cb} \\ 0 & 1 \end{bmatrix}, \quad (13)$$

where R_b is the rotation matrix of enclosed ellipsoid to the mobile platform, and the following equations can be written:

$$\vec{X}_{cb} = \begin{bmatrix} X_{cb} \\ Y_{cb} \\ Z_{cb} \end{bmatrix} = \begin{bmatrix} X \\ Y \\ \frac{l_{b2}}{2} \end{bmatrix}, \quad (14)$$

$$R_b(\phi) = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (15)$$

where the potential functions for collision avoidance between mobile platform and obstacles are calculated by Eq. (9).

Also, the global equation of enclosed ellipsoid to the first link of Scout robot can be presented, using the transformation matrix A_{l1} between the local system attached to the first link and the reference system, and is equal to:

$$A_{l1}^{-1} = \begin{bmatrix} (R_b R_{l1})^T & - (R_b R_{l1})^T \vec{X}_{cl1} \\ 0 & 1 \end{bmatrix}, \quad (16)$$

where \vec{X}_{cl1} (which is the replacement vector of the center of the first link in the reference coordinate systems) and the rotation matrix R_{l1} are presented as follows:

$$\vec{X}_{cl1} = \begin{bmatrix} X_{cl1} \\ Y_{cl1} \\ Z_{cl1} \end{bmatrix} = \begin{bmatrix} X + f \sin(\phi) \\ Y - f \cos(\phi) \\ l_{b2} + 0.5 l_1 \end{bmatrix}, \quad (17)$$

$$R_{l1}(\theta_1) = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (18)$$

By following the same approach for the second link of Scout manipulator, the global equation of fitted ellipsoid to the second link can be written via matrix A_{l2} :

$$A_{l2}^{-1} = \begin{bmatrix} (R_b R_{l1} R_{l2})^T & - (R_b R_{l1} R_{l2})^T \vec{X}_{cl2} \\ 0 & 1 \end{bmatrix}, \quad (19)$$

where \vec{X}_{cl2} and A_{l2} are shown as follows:

$$\begin{aligned} \vec{X}_{cl2} &= \begin{bmatrix} X_{cl2} \\ Y_{cl2} \\ Z_{cl2} \end{bmatrix} \\ &= \begin{bmatrix} X + f \sin(\phi) + 0.5 l_2 \cos(\phi + \theta_1) \cos(\theta_2) \\ Y - f \cos(\phi) + 0.5 l_2 \sin(\phi + \theta_1) \cos(\theta_2) \\ l_{b2} + l_1 + 0.5 l_2 \sin(\theta_2) \end{bmatrix}, \end{aligned} \quad (20)$$

$$R_{l2} = \begin{bmatrix} \cos(-\theta_2) & 0 & \sin(-\theta_2) \\ 0 & 1 & 0 \\ -\sin(-\theta_2) & 0 & \cos(-\theta_2) \end{bmatrix}. \quad (21)$$

4. Simulation and experimental results

Herein, some simulations and experimental results of optimal trajectory planning of Scout robot are presented. It must be mentioned that regarding the simulation and software limitation, two joints of the right arm of Scout manipulator are used for this study, and the others are fixed. The characteristics values of Scout parts are shown in Table 1.

Furthermore, the torque-speed characteristics of the DC actuators of the Scout mobile manipulator are equal to:

$$\begin{aligned} U^+ &= K_1 - K_2 \dot{\phi}, \\ U^- &= -K_1 - K_2 \dot{\phi}, \end{aligned} \quad (22)$$

where:

$$U^+ = [\tau_{r,\max} \quad \tau_{l,\max} \quad \tau_{1,\max} \quad \tau_{2,\max}]^T,$$

$$U^- = [\tau_{r,\min} \quad \tau_{l,\min} \quad \tau_{1,\min} \quad \tau_{2,\min}]^T,$$

$$\dot{\phi} = [\dot{\theta}_r \quad \dot{\theta}_l \quad \dot{\theta}_1 \quad \dot{\theta}_2]^T,$$

where the actuator constants are given as follows:

$$\begin{aligned} K_1 &= [6.85 \quad 6.85 \quad 0.55 \quad 0.96]^T \text{ N m}, \\ K_2 &= [0.4956 \quad 0.4956 \quad 0.094 \quad 0.183] \text{ N ms/rad}. \end{aligned} \quad (23)$$

To verify the ability of the presented method for motion planning of wheeled mobile robot in the presence of multiple obstacles, at the first simulation, Scout manipulator must move from the initial state $P_0(X = 0 \text{ m}, Y = 0 \text{ m}, \phi = 0 \text{ rad}, \theta_1 = -\pi \text{ rad}, \theta_2 = -\pi/6 \text{ rad})$ to the final state $P_f(X = 1.5 \text{ m}, Y = -0.4 \text{ m}, \phi = 0 \text{ rad}, \theta_1 = -\pi/4 \text{ rad}, \theta_2 = \pi/4 \text{ rad})$ at $t_f = 3 \text{ s}$. Also, there are an ellipsoid obstacle and a sphere obstacle in the test environment. The center point and the radii of the ellipsoid are $P_1(X_{cob1} = 0.4 \text{ m}, Y_{cob1} = -0.15 \text{ m}, Z_{cob1} = 0.1 \text{ m})$ and $(a_{ob1} = 0.3 \text{ m}, b_{ob1} = 0.1 \text{ m}, c_{ob1} = 0.05 \text{ m})$. The center point of sphere is $P_2(X_{cob2} = 0.2 \text{ m}, Y_{cob2} = -0.6 \text{ m}, Z_{cob2} = 0.2 \text{ m})$ and its radius is $r_{ob2} = 0.1 \text{ m}$. In addition, weighting matrix of the state vector is presumed as $W =$

Table 1: Parameters of Scout mobile manipulator.

Parameter	Value	Unit
Length of Links	$l_1 = 0.16, l_2 = 0.21$	m
Mass of Links	$m_1 = 0.128,$ $m_2 = 0.231$	kg
Moment of inertia of link 1	$J_1 = 0.00005$	kg m ²
Moment of inertia of link 2	$J_{2x} = 0.00008$ $J_{2y} = 0.00091$ $J_{2z} = 0.00092$	kg m ²
Mass of base	$m_0 = 6.0$	kg
Mass of wheels	$m_w = 0.32$	kg
Moment of inertia of the base about Z axis	$J_l = 0.06363$	kg m ²
Moment of inertia of the wheels about rotation axis	$J_w = 0.0008$	kg m ²
e	0.145	m
r	0.08	m
d	0.065	m
f	0.08	m
l_{b1}	0.06	m
l_{b2}	0.21	m

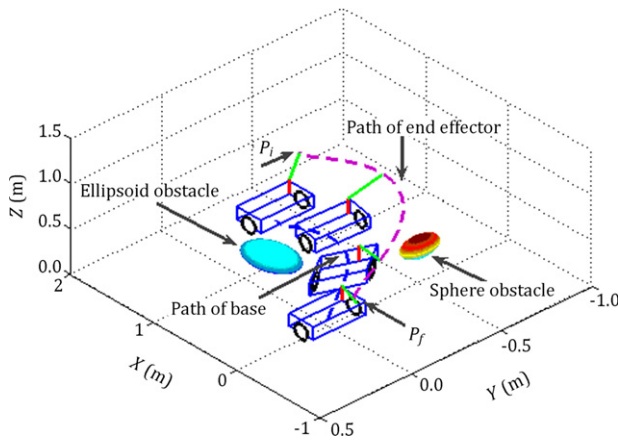


Figure 5a: Optimal trajectory of Scout robot (isometric view).

$(0 \quad \dots \quad 0 \quad 0.1 \quad 0.1 \quad 0.1 \quad 0.1)_{11 \times 1}$, and weighting matrix of the control input is $R = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$. The weightings values of collision avoidance are assumed as:

$$\begin{aligned} w_{ob1b} &= 0.2, & w_{ob1l_1} &= 0.2, & w_{ob1l_2} &= 0.2, \\ w_{ob2b} &= 0.2, & w_{ob2l_1} &= 0.2, & w_{ob2l_2} &= 0.2. \end{aligned}$$

The optimal path of Scout robot is shown in Figures 5a and 5b. It must be noticed that in Figure 5b, the optimal path of the mobile platform and the end effector of Scout robot without existing any obstacles, and also considering multiple obstacles in cluttered environment are compared.

As seen in Figure 5a, the optimal path of mobile manipulator in presence of ellipsoid obstacle and sphere obstacle is presented, and Scout manipulator avoids obstacles in its point-to-point motion. Furthermore, Figure 5b compares the optimal path of the mobile robot in the presence of multiple obstacles with a non-obstacle environment. If the weighting coefficients of potential functions are assumed to be zero, the mobile robot collides with obstacles. The simulation results clearly show the ability of the method for trajectory planning of mobile manipulator without colliding with the obstacles. Also, the optimal torques exerted to wheels and the joints of the mobile robot are depicted in Figures 6a–6d. As seen, if there are not any obstacles in the environment, the workspace of the robot is more extended and the constraint of obstacle avoidance is not

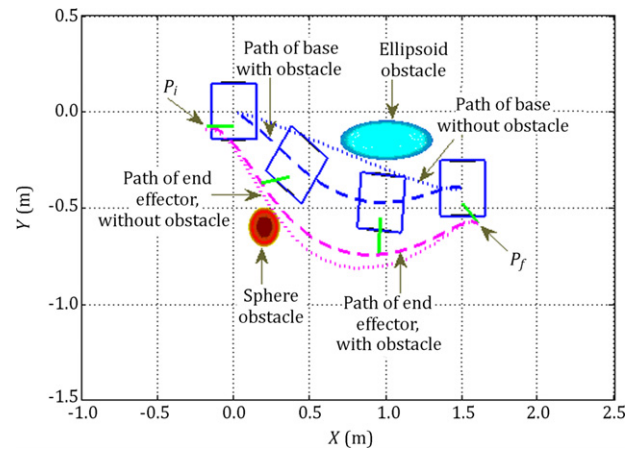


Figure 5b: Optimal trajectory of Scout manipulator (upper view).

existed. But if the mobile robot moves among the obstacles, the collision-free motion of mobile manipulator causes the relative increase of the actuators torques. Table 2 shows the maximum torque values, and compares the effect of obstacle on torque values of the actuators. It is seen that the existence of obstacles increases the maximum values of actuators torque, especially for ones which are closer to the obstacles. For example, the maximum torque values of the actuators of left wheel and second joint are exceeded more, because these are closer to the obstacles than the other actuators.

In fact, by changing the relative value of the weighting coefficients, the various optimal trajectories are obtained with different characteristics and the path designer can select a suitable path among the numerous optimal trajectories. For another simulation, the relative changing of weighting coefficients of potential functions and its effect on generated paths is discussed. The Scout manipulator moves from the initial state $P_0(X = 0 \text{ m}, Y = 0 \text{ m}, \varphi = \frac{\pi}{4} \text{ rad}, \Theta_1 = -\pi \text{ rad}, \Theta_2 = -\pi/12 \text{ rad})$ to the final state $P_f(X = 1 \text{ m}, Y = 1 \text{ m}, \varphi = \frac{\pi}{4} \text{ rad}, \Theta_1 = -\pi/4 \text{ rad}, \Theta_2 = \pi/4 \text{ rad})$. It must be noticed that the mobile manipulator operates in a common workspace and the overall time of motion is predefined by the path planner. So, the movement time is presumed as a specified parameter and determined by the designer. Herein, the overall time of robot motion is predefined as $t_f = 3.8 \text{ s}$. Also, there is

Table 2: Maximum values of the actuator torques.

	The right wheel (N m)	The left wheel (N m)
Environment without obstacles	1.0608	2.6476
Environment with obstacles	1.4898	4.8754
	The first joint (N m)	The second joint (N m)
Environment without obstacles	0.2814	0.5010
Environment with obstacles	0.4271	0.7038

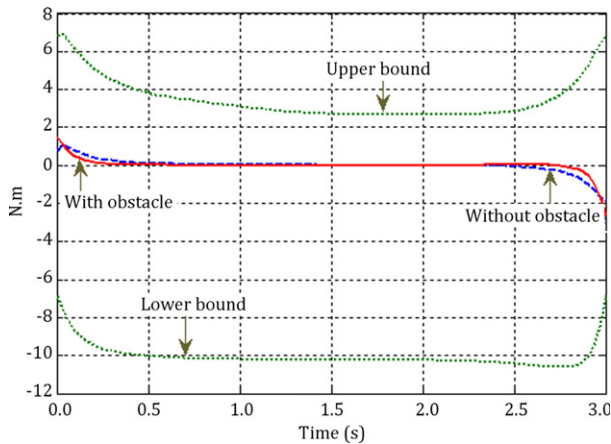


Figure 6a: Optimal torque exerted to the right wheel.

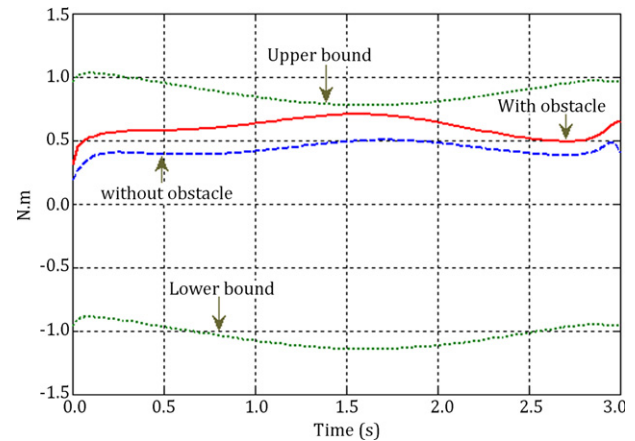


Figure 6d: Optimal torque exerted to the second joint.

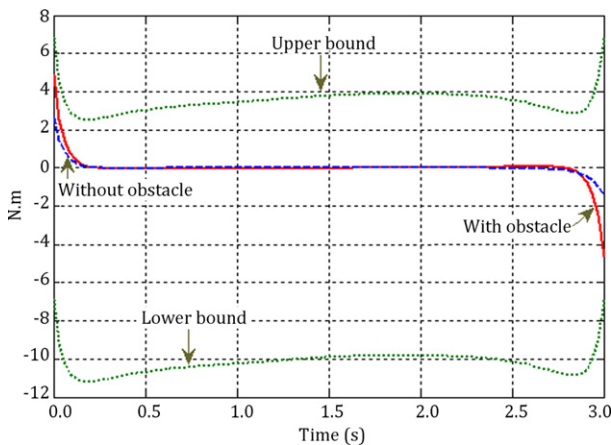


Figure 6b: Optimal torque exerted to the left wheel.

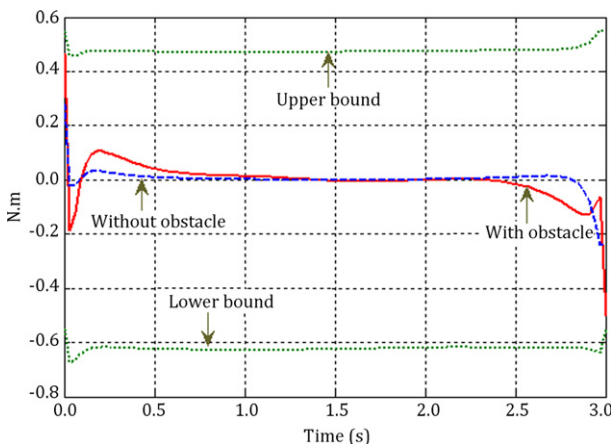


Figure 6c: Optimal torque exerted to first joint.

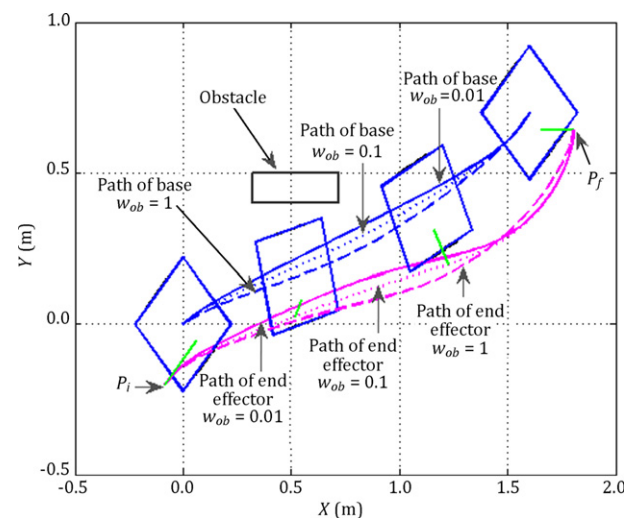


Figure 7: Optimal path of the Scout robot (upper view).

a cuboid obstacle with the center point coordinate $P_1(X_{cob1} = 0.52 \text{ m}, Y_{cob1} = 0.45 \text{ m}, Z_{cob1} = 0.25 \text{ m})$ and the dimensions $2l = 0.4 \text{ m}, 2k = 0.1 \text{ m}, 2h = 0.5 \text{ m}$ in the test environment. In addition, weighting matrix of the state vector is $W = \begin{pmatrix} 0 & \dots & 0 & 0.1 & 0.1 & 0.1 & 0.1 \end{pmatrix}_{11 \times 1}$, and weighting matrix of the control input is $R = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$. In this simulation, the obstacle weightings are assumed as $w_{ob1b}, w_{ob1l1}, w_{ob1l2} = 0.01$ or 0.1 or 1 , and the effect of changing of the weighting coefficients on the optimal path is considered. The optimal path of Scout robot is shown in Figure 7 and the optimal velocities of the wheels and joint are depicted in Figures 8a–8d.

According to Figure 7, by increasing the weighting coefficients of potential functions, the minimum distance from the mobile manipulator to the obstacle is increased, hence the max-

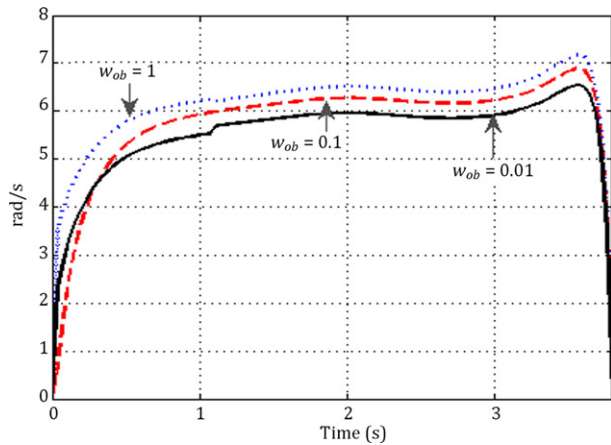


Figure 8a: Optimal velocity exerted to the right wheel.

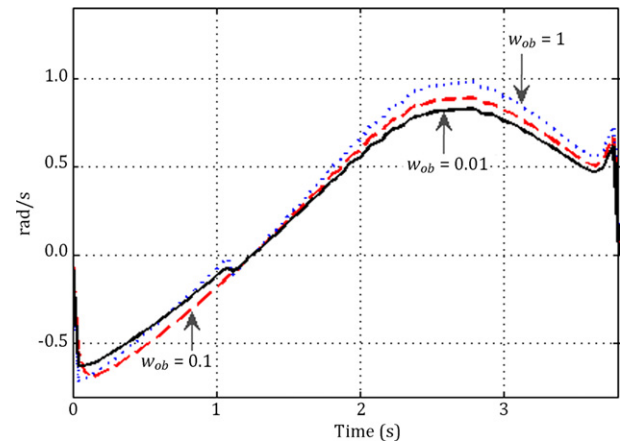


Figure 8d: Optimal velocity exerted to the second joint.

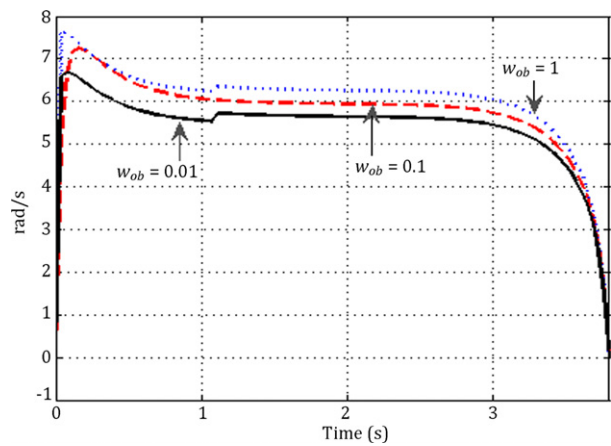


Figure 8b: Optimal velocity exerted to the left wheel.

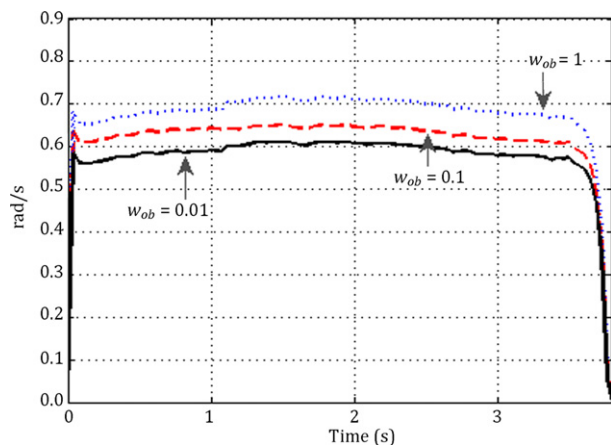


Figure 8c: Optimal velocity exerted to the first joint.

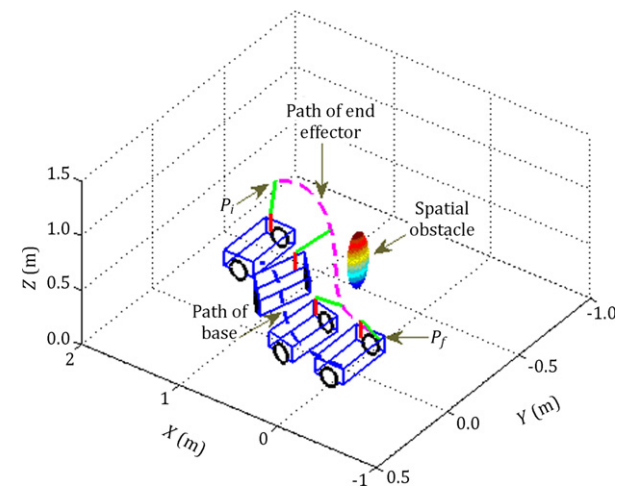


Figure 9a: Optimal trajectory of the Scout robot (isometric view).

imum values of the velocities of the wheels and joints increase, too. So, it is a trade-off between safety and characteristics of optimal paths, and the path designer can select a suitable path among some generated optimal paths.

For the last simulation, it is desired to investigate the feasibility of optimal trajectories generated via the presented method. So, the optimal motion of the Scout manipulator is simulated and compared with the experimental results. For this simulation study, the initial and final state of the robot

motion, the weighting matrices and weighting coefficients and the final time of motion are the same with the first simulation, but there is the ellipsoidal obstacle with center point ($X_{cob1} = 0.5$ m, $Y_{cob1} = -0.4$ m, $Z_{cob1} = 0.4$ m) and the radii ($a_{ob1} = 0.05$ m, $b_{ob1} = 0.05$ m, $c_{ob1} = 0.25$ m). Figures 9a and 9b show the simulations of the optimal path, and Figures 10a and 10b depict the angular velocities of Scout manipulator. As seen in Figures 9a and 9b, the mobile manipulator avoids colliding with spatial obstacle in its optimal path. In addition, a comparison between simulation and experimental results of the angular displacement of wheels and joints of Scout robot is depicted in Figures 11a–11d.

As seen, the simulation results can be followed by experimental tests. Therefore, simulation and experimental results are nearly the same, and the collision of Scout manipulator with obstacles is avoided. This fact illustrates that the generated optimal trajectories are feasible and can be used in application. Also, it should be noticed that a little difference between experiments and simulations can be resulted from errors of the wheels and joints encoders, and errors of sensors and transformation data delays from the robot sensors to its computer. So, the distance difference between simulation and experiment for the final position of mobile base is 0.056 m, and the distance difference between simulation and experiment for the final position of end effector is 0.055 m. Figure 12 compares the simulated optimal path of Scout robot with the experimental results.

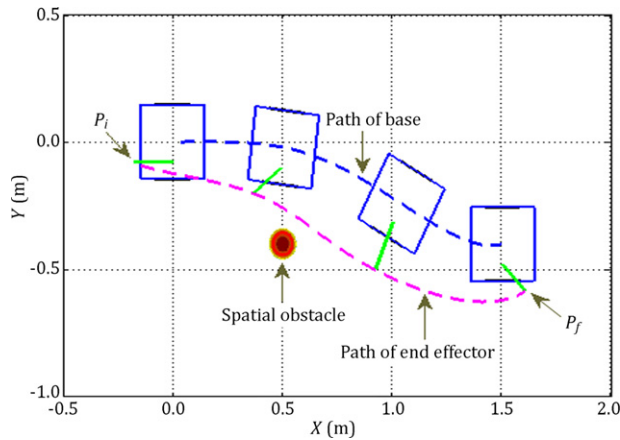


Figure 9b: Optimal trajectory of the Scout robot (upper view).

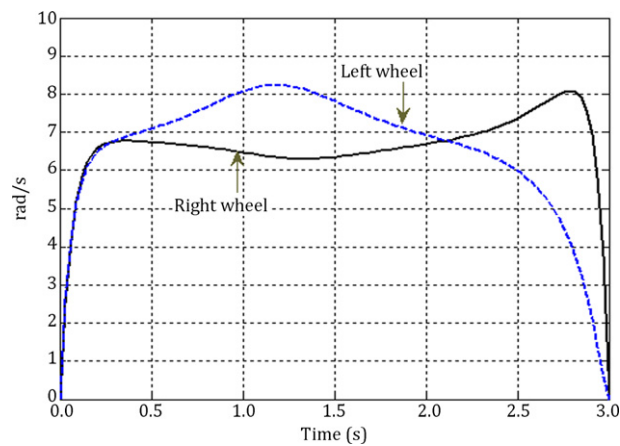


Figure 10a: Optimal velocities of the wheels.

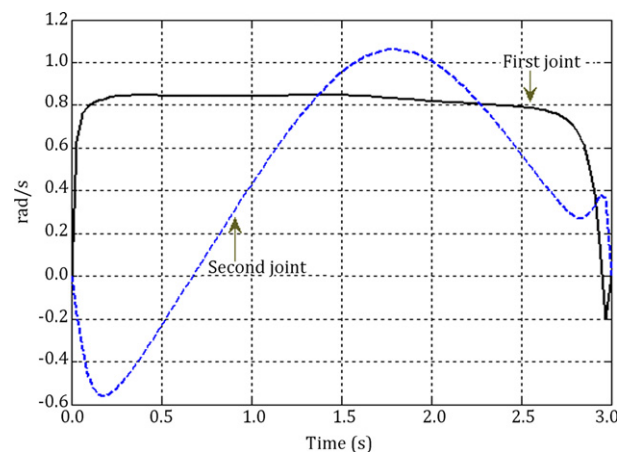


Figure 10b: Optimal velocities of the joints.

5. Conclusion

In this paper, the trajectory planning of wheeled mobile manipulators in cluttered environments has been presented using the appropriate potential functions. By modeling the obstacles and mobile manipulator as ellipsoids, the transformation matrix between global and local coordinate systems has been employed to represent ellipsoids equations in the refer-

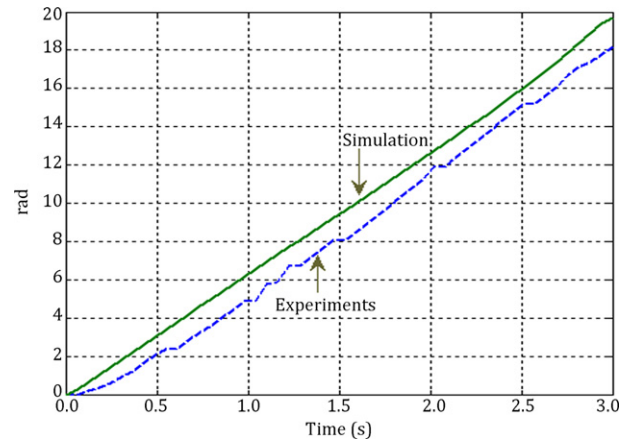


Figure 11a: Angular displacement of the right wheel.

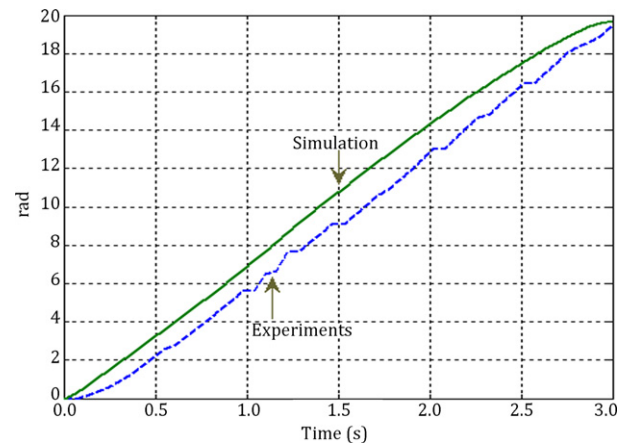


Figure 11b: Angular displacement of the left wheel.

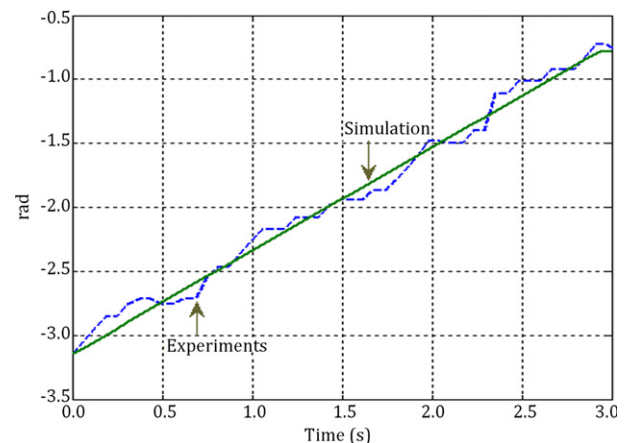


Figure 11c: Angular displacement of the first joint.

ence coordinate system. For the collision avoidance of the enclosed ellipsoids, the dimensionless potential functions have been defined. The full nonlinear dynamics equations of mobile robot have been derived and the trajectory planning of mobile robot in cluttered environment has been done. According to a simple formulation of potential functions, some simulations and experiments for Scout mobile manipulator in the three-dimensional space have been performed. The optimal path and maximum torque values of the actuators for non-obstructed en-

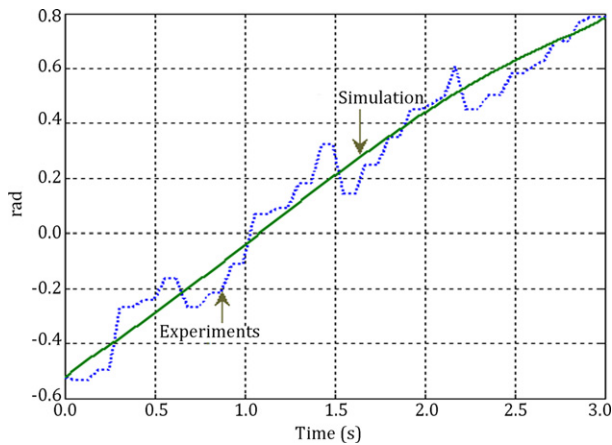


Figure 11d: Angular displacement of the second joint.

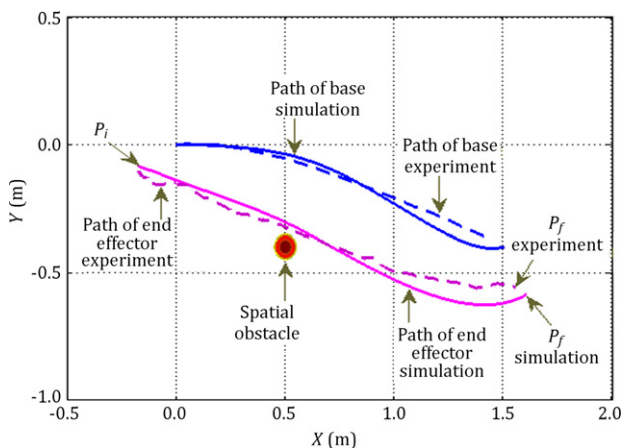


Figure 12: Optimal path of the Scout robot (upper view).

environment and in the presence of multiple obstacles have been compared, and it has been shown that obstacle avoidance has exceeded maximum values of control input. In addition, the simulations and experimental results have demonstrated simplicity of the presented formulation and applicability of the proposed method.

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